## Commutator theory for rack and quandles

*Quandles* are idempotent lef-distributive left-quasigroups and they arise in different areas of mathematics as knot theory, the study of solutions of the set theoretic Yang-Baxter equation, braiding vector spaces and Nichols algebras.

Group and module theory has been used to investigate quandles exploiting the strong interplay between the properties of a quandle Q and the group-theoretical properties of its *displacement group*.

In a recent paper we adapt commutator theory for general algebras (in the sense of Freese, McKenzie [FM87]) to the setting of quandles [BS19]. We proved that properties as abelianness and centrality of congruences are completely characterized by the properties of the displacement group and its subgroups. Moreover there exists a Galois connection between the congruence lattice of a quandle and a sublattice of the normal subgroups of the displacement group, which can be exploited to get information on the displacement group from its congruence lattice and vice versa.

These techniques are particularly useful for connected quandles, which admits a representation over their displacement groups. Some new results based on this theory are the classification of connected quandles of size  $p^3$  [BB19] and of size pq and 4p (in preparation) where p and q are primes and the characterization of doubly homogenous quandles [Bon19].

## References

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