

# Quadratic Forms and Differential Operators

## Professor

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## Summary

It is the aim of this lecture to present Kato's First and Second Representation Theorem in full details. This is a classical tool to derive Friedrich's extension of a symmetric operator and it is a widely used tool for partial differential equations. In particular we will show its importance in practical problems when we discuss the Laplace operator in higher dimensions with different classes of potentials.

## Contents

- **Chapter 1: Unbounded operators**

Closedness, closability, self-adjointness, spectrum, differential operators, further examples.

- **Chapter 2: Sesquilinear forms in Hilbert spaces and the associated operators**

Definitions, semiboundedness, closed forms, closable forms, relative boundedness for forms and operators

- **Chapter 3: Representation theorem**

First representation theorem, Friedrichs extension, other examples for the representation theorem, second representation theorem, polar decomposition of a closed operator.

- **Chapter 4: Perturbation theory for sesquilinear forms and the associated operators**

Real part of an  $m$ -sectorial operator, perturbation of an  $m$ -sectorial operator and its resolvent, symmetric unperturbed operators, pseudo-Friedrichs extensions.

- **Chapter 5: Quadratic forms and the Schrödinger operators**

Ordinary differential operators, Dirichlet form and Laplace operator, Schrödinger operators in  $\mathbb{R}^3$ .

## References

- [1] T. Kato, *Perturbation Theory for Linear Operators*, Springer, New York, 1966.