Christopher Andrew Hooley

Plan of lectures

Course title: "Frustrated magnetism and spin liquids"

Syllabus: Types of frustration; quantifying the degree of frustration. Consequences of frustration in classical models of magnetism; non-collinear and non-coplanar order. Frustration in quantum models of magnetism; failure of the spin-wave expansion. One-dimensional examples; the Δ-chain and its spinons. Short-range-ordered states; valence bond crystals; entanglement, and the connection to tensor networks.

Lecture-by-lecture plan:

1. What is frustration, and how do we quantify it?
    a. Types of frustration:
        i. geometrical frustration;
        ii. mixed ferromagnetic and antiferromagnetic interactions;
        iii. further-neighbour interactions;
        iv. Dzyaloshinskii-Moriya interactions.
    b. Quantifying the degree of frustration:
        i. the ratio between Curie-Weiss temperature and ordering temperature;
        ii. the ratio between the maximum and minimum of the structure factor;
        iii. the number of free angles per spin in a classical model.
2. Geometrical frustration and its consequences.
    a. The classical Heisenberg antiferromagnet on the d=2 triangular lattice:
        i. calculation of the structure factor;
        ii. non-collinear magnetic order;
        iii. the 'zero-spin-triangle' reformulation;
        iv. free-angle count.
    b. The classical Heisenberg antiferromagnet on the d=3 pyrochlore lattice:
        i. the 'zero-spin-tetrahedron' reformulation;
        ii. free-angle count;
        iii. non-coplanar magnetic order.
3. Competing-interaction frustration and its consequences.
    a. The J_1-J_2 model:
        i. Hamiltonian.
        ii. A limiting case: $J_2/J_1 \to 0$.
        iii. Another limiting case: $J_1/J_2 \to 0$.
        iv. Structure factor for arbitrary J_1/J_2.
        v. Phase diagram of the classical J_1-J_2 model.
    b. The Heisenberg ferromagnet with additional Dzyaloshinskii-Moriya terms:
        i. Hamiltonian.
        ii. Structure factor.
        iii. Phase diagram.
4. Quantum fluctuations:

a. The Heisenberg antiferromagnet on the d=2 triangular lattice:
    i. Spin-wave spectrum.
    ii. Correction to zero-temperature ordered moment.
b. The Heisenberg antiferromagnet on the d=2 kagome lattice:
    i. Spin-wave spectrum.
    ii. Divergence of moment correction: failure of spin-wave expansion.

5. If not long-range order, then what?
    a. The Δ-chain:
        i. Valence bond crystal ground states.
        ii. Domain walls: two types of spinon.
        iii. Excitation spectrum.
    b. The columnar-dimer model:
        i. Phase diagram of the classical Heisenberg model.
        ii. Quantum model: Néel antiferromagnet and valence bond crystal.
    c. The Rokhsar-Kivelson point.
6. Unsolved problems and connections to other fields:
    a. Resonating valence bond states and high-T_c superconductivity.
    b. Gapless spin liquids, Z_2 gauge theories, and all that.
    c. Entanglement, matrix-product states, and tensor networks.
    d. Spin liquids in experiment.